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LIE'S VIEWS ON SEVERAL IMPORTANT POINTS IN MODERN MATHEMATICS.

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It is generally admitted that America has contributed comparatively little towards the advancement of the science of mathematics. During the last twenty years there has been a rapidly increasing progress in this direction. Several European countries have also moved forward at a rapid rate during this period, so that our relative position is not improving as rapidly as might be desired.

The standard of general scholarship required for the higher degrees at our better institutions is comparatively high but the number of important discoveries does not yet correspond to this standard. In fact, the two are not apt to advance very far together, for the field of mathematics is so extensive that most are compelled to choose between a superficial acquaintance with the whole range of mathematical research and an exhaustive knowledge of only a few subjects.

In view of these facts it is natural that there should be many who strive to lead American mathematical talent to those newer regions which seem to offer the most fruitful fields of investigation. While there is a great difference of opinion with respect to these regions yet the most successful investigators are in the best possible position to judge in regard to them.

The view expressed by Klein during last year, in his address on *Arithmetizing Mathematics*, that Lie in Leipzig, Germany, and Poincaré in Paris, France, are the two most active mathematical investigators of the present day, is quite generally held. The following translation of a part of the introductory remarks of an article* published during last year by the former of these may therefore be

*Berichte der Koenigl. Saechs. Gesellschaft, 1895.

of considerable interest, as it contains the views of the author in regard to several important points in mathematics, especially in regard to the most important newer regions.

"In this century the concepts known as substitution and substitution group, transformation and transformation group, operation and operation group, invariant, differential invariant, and differential parameter, appear continually more clearly as the most important concepts of mathematics. While the curve as the representation of a function of a single variable has been the most important object of mathematical investigation for nearly two centuries from Descartes, while on the other hand, the concept of transformation first appeared in this century as an expedient in the study of curves and surfaces, there has gradually developed in the last decades a general theory of transformations whose elements are presented by the transformation itself while the series of transformations, in particular the transformation groups, constitute the object.

The general theory of transformations is a branch of analysis in the sense that it can be developed by purely analytic methods. It has however the material geometrical property that its operations are not only conceivable but directly intuitive to a large extent.

If we consider that the difference between the analytic and the synthetic methods exists in the fact that the synthesist reasons with concepts while the analyst operates with symbols, according to fixed rules, we may see an important property of the theory of transformations in this that its theorems can be developed in an elegant analytic as well as in a perspicuous even intuitively clear manner. It is due to this fact that the theory of transformations is considerably simpler than the theory of substitutions.

It should be added that different branches of mathematics have contributed to the development of the theory of transformations and that many parts of mathematics have already been considerably advanced by means of this theory.

The theory of differential equations is the most important branch of mathematics. Each department of physics presents problems which depend upon the integration of differential equations. In general, the theory of differential equations involves the road towards the explanation of all natural phenomena which require time. While this theory has an infinite practical value it has also a corresponding theoretic importance since it leads in a rational manner to the study of new important functions and classes of functions."

Gottingen, Germany, October 26, 1896.